STRING SOLITONS AND BLACK HOLE THERMODYNAMICS

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We discuss the role of string solitons in duality and examine the feature of compositeness, which allows for the interpretation of general solutions as bound states of supersymmetric fundamental constituents. This feature lies at the heart of the recent success of string theory in reproducing the Bekenstein-Hawking black hole entropy formula. Talk given at 19th annual MRST meeting, Syracuse, NY, May 12-13, 1997. McGill 97-20.

The standard model of elementary particle physics has proven very succesful in describing three of the four fundamental forces of nature. In the most optimistic scenario, the standard model can be generalized to take the form of a grand unified theory (GUT), in which quantum chromodynamics (QCD), describing the strong force, and the electroweak theory, unifying the weak interaction with electromagnetism, are synthesized into a single theory in which all three forces have a common origin.

The framework for studying these three forces is that of Yang-Mills gauge theory, a certain class of quantum field theory based on the principle of gauge symmetry. In any quantum-mechanical theory, the natural length scale associated with a particle of mass m (such as an elementary particle) is given by the Compton wavelength $\lambda_C = \hbar/mc$, where $\hbar$ is Planck's constant divided by 2π and c is the speed of light. Scales less than λ_C are therefore unobservable within the context of the quantum mechanics of this elementary particle.

Quantum field theory, however, has so far proven unsuccessful in describing the fourth fundamental force, namely gravitation. The succesful framework in this case is that of general relativity, which, however, does not seem to lend itself to a straightforward attempt at quantization. The main problem in such an endeavour is that the divergences associated with trying to quantize gravity cannot be circumvented (or "renormalized") as they are for the strong, weak and electromagnetic forces.

Among the most interesting objects predicted by general relativity are black holes, which represent the endpoint of gravitational collapse. According to relativity, an object of mass m under the influence of only the gravitational force (*i.e.* neutral with respect to the other three forces) will collapse into a region of spacetime bounded by a surface, the event horizon, beyond which signals cannot be transmitted to an outside observer. The event horizon for the simplest case of a static, spherically symmetric black hole is located at a

radius $R_S = 2Gm/c^2$, the Schwarzschild radius, from the collapsed matter at the center of the sphere, where G is Newton's constant.

In trying to reconcile general relativity and particle physics, even at an intuitive level, a natural question to ask is whether they have a common domain. This would arise when an elementary particle exhibits features associated with gravitation, such as an event horizon. This may occur provided $\lambda_C \lesssim R_S$, which implies that, even within the framework of quantum mechanics, an event horizon for an elementary particle may be observable. Such a condition is equivalent to $m \gtrsim m_P = \sqrt{\hbar c/G} \sim 10^{19} GeV$, the Planck mass, or $\lambda_C \lesssim l_P = \sqrt{\hbar G/c^3}$, the Planck scale. It is in this domain that one may study a theory that combines quantum mechanics and gravity, the so-called *quantum gravity* (henceforth we use units in which $\hbar = c = 1$).

A problem, however, arises in this comparison, because most black holes are thermal objects, and hence cannot reasonably be identified with pure quantum states such as elementary particles. In fact, in accordance with the laws of *black hole thermodynamics*[1,2] black holes radiate with a (Hawking) temperature constant over the event horizon and proportional to the surface gravity: $T_H \sim \kappa$. Furthermore, black holes possess an entropy $S = A/4G$, where A is the area of the horizon (the area law), and $\delta A \geq 0$ in black hole processes. Finally, the energy law of black hole thermodynamics takes the form $dM = (\kappa/8\pi)dA + \Omega_H dJ$, in analogy with $dE = TdS - PdV$, where in the former case Ω_H is the angular velocity of the horizon and J the angular momentum of the black hole.

In this framework, then, a pure state with $S = 0$ such as an (electrically charged) elementary particle corresponds to a black hole with zero area. Such black holes arise as extremal limits of two-parameter (mass and charge) families of charged black holes. From the cosmic censorship principle, which forbids the existence of "naked" singularities (*i.e.* singularities not hidden behind a horizon), such solutions are required to satisfy a bound between the mass M and charge Q, *e.g.*, $M \geq Q$. Alternatively, this bound can be expressed in terms of the outer (event) horizon R_+ and inner horizon R_- of the black hole (the latter horizon representing the limit of energy extraction from the black hole): $R_+ \geq R_-$. Extremality then represents the saturation of the bound by $M = Q$ or $R_+ = R_-$. We shall see later that, while some extremal black holes possess zero area, and therefore zero entropy, others do not. The former can potentially correspond to elementary particles (pure states), while the latter correspond to an *ensemble* of particles or states.

At the present time, string theory, the theory of one-dimensional extended

objects, is the only known reasonable candidate theory of quantum gravity. The divergences inherent in trying to quantize point-like gravity seem not to arise in string theory. Furthermore, string theory has the potential to unify all four fundamental forces within a common framework.

The two-dimensional worldsheet swept out by a string is embedded in a higher-dimensional (target) spacetime, which in turn represents a background for string propagation. At an intuitive level, one can see how point-like divergences may possibly be avoided in string theory by considering the four-point amplitudes arising in string theory [3]). Unlike those of field theory, the four-point amplitudes in string theory do not have well-defined vertices at which the interaction can be said to take place, hence no corresponding divergences associated with the zero size of a particle. A simpler way of saying this is that the finite size of the string smooths out the divergence of the point particle.

The ground states of string theory correspond to conformal invariance of the two-dimensional sigma model of a genus zero (sphere) worldsheet. Solving the beta-function equation of this sigma-model then corresponds to classical solutions of string theory. Within this classical theory, the perturbative parameter is $\alpha' = 1/(2\pi T_2) = l_s^2$, where T_2 is the tension of the string and l_s is the string length.

Perturbative quantum corrections in string theory take the form of an expansion in the genus of the worldsheet, with coupling parameter $g = \exp\phi$, where ϕ, the dilaton, is a dynamical scalar field.

Consistent, physically acceptable string theories possess supersymmetry between bosons and fermions, and supersymmetric string theory requires a ten-dimensional target space. This leads to another feature of string theory, namely, compactification, *i.e.*, the splitting of the ten dimensional vacuum into the product of a four-dimensional vacuum and a compact six-dimensional manifold which may be shrunk to a point.

Finally, the property of string theory that is of most interest to us in these lectures is that of *duality*. At the simplest level, duality is a map that takes one theory into another theory (or possibly the same theory in a different domain). An immediate consequence of duality is that the two theories are physically equivalent in the appropriate domains. It then follows that calculations performed on one side can be immediately carried over into the other, even if direct calculations in the latter theory may have previously been intractable.

The two most basic dualities in string theory are the target space T-duality [4] and the strong/weak coupling S-duality [5]. Suppose in a compactification one of the dimensions of the six-dimensional compactification manifold is wrapped around a circle with radius R. Then T-duality is a generalization of a map that takes a string theory with radius R into a theory with radius $\alpha'/R = l_s^2/R$.

This implies that a radius smaller than the string scale is equivalent to a radius larger than the string scale. Effectively, then, the string scale is a minimal scale, which conforms to our previous intuition that the size of the string smooths out the point-like divergence. T-duality is a classical, worldsheet duality and in various compactifications has been shown to be an exact duality in string theory.

S-duality, by contrast, is a quantum (string loop), spacetime duality and generalizes the map that takes the string coupling constant g to its inverse $1/g$. Such a map takes the weak coupling domain into the strong coupling domain within a given string theory and allows us to use perturbative results in the latter. Also unlike T-duality, S-duality has only been established exactly in the low-energy limit of string theory.

These two dualities and the interplay between them are at the heart of the recent activity in string theory. This activity has also been fueled by the realization that perturbative string theory is insufficient to answer the most fundamental questions of string theory, such as vacuum selection, supersymmetry breaking, the cosmological constant problem and, finally, the problem of understanding quantum gravity from string theory. All these questions require *nonperturbative* information.

What kind of objects arise in nonperturbative physics? Solitons, or topological defects[6], are inherently nonperturbative solutions, representing objects with mass $m_s \sim 1/g^2$, where g is the coupling constant of the theory. Examples of solitons are magnetic monopoles or domain walls. The connection between duality and solitons often involves the interchange of perturbative, fundamental (electric) particles with nonperturbative, solitonic (magnetic) objects. This is the main feature of the Montonen-Olive conjecture[7] for $N = 4$, $D = 4$ supersymmetric Yang-Mills gauge theory, which postulates the existence of a dual version of the theory in which electric gauge bosons and magnetic monopoles interchange roles. In this scenario, the monopoles become the elementary particles and the gauge bosons become the solitons.

What does a duality map look like? Let us look at the simplest example. Consider four-dimensional point-like electromagnetism, a $U(1)$ gauge theory with gauge field A_M and field strength $F_{MN} = \partial_M A_N - \partial_N A_M$. The field of an electric charge Q_e located at the origin is given by $E_r = F_{0r} = Q_e/r$, where r is the radial coordinate. The field of a magnetic charge Q_m located at the origin is given by $B_r = F_{\theta\phi} = Q_m/r$, where θ annd ϕ are coordinates on the two-sphere S^2. Now the *dual* of the field strength is given by $\tilde{F}_{AB} = (1/2)\epsilon_{ABCD}F^{CD}$, where ϵ_{ABCD} is the four-dimensional Levi-Civita tensor. This map is easily generalized to an arbitrary number of dimensions. In all cases, the dual of the dual of a tensor reproduces the original tensor up to a sign: $\tilde{\tilde{F}} = \pm F$. For

our four-dimensional example, $\tilde{F}_{0r} = F_{\theta\phi}$. So under the map that takes F to $\tilde{F}$, there is an interchange of Q_e and Q_m. Another way of saying this is that, in the dualized version of electromagnetism, what had previously appeared as electric charge now appears as magnetic and vice-versa.

Now consider a one-dimensional extended object, a string. In analogy with the point-particle, the string couples to an antisymmetric gauge field, but this time in the form of a two-tensor B_{MN}, with three-form field strength H_{MNP}. For the dual of H to represent the field strength of a string, the Levi-civita tensor must be six-dimensional: $\tilde{H}_{ABC} = (1/6)\epsilon_{ABCDEF}H^{DEF}$. So a duality between string theories is most naturally formulated in six dimensions, where the "electric" string charge read from H_{01r} (where x^1 is the direction of the string) is interchanged with the "magnetic" string charge read from $H_{\chi\theta\phi}$, where χ, θ and ϕ are coordinates on the three-sphere S^3.

In the low-energy limit, the main feature of string/string duality is the following: in one version of string theory, there exists an electric, elementary string solution corresponding to a perturbative state of the theory and a magnetic, solitonic string solution corresponding to a nonperturbative state. In the dual string theory, the solution that appears electric in the first version now appears magnetic and vice-versa, while the state that appears perturbative in the first version now appears nonperturbative and vice-versa. The duality map relates the string coupling g of the first version to that of its dual g' via $g = 1/g'$, so that the weak and strong coupling regimes of the two theories are interchanged. A interesting and nontrivial consequence of string/string duality is that, in compactifying down to four dimensions, the duality map takes the spacetime, strong/weak coupling S-duality of one version into the worldsheet, target space T-duality of the dual version. Since T-duality is in many cases established as exact, the conjectured S-duality would then follow as a consequence of string/string duality.

For the purpose of understanding black hole thermodynamics from string theory, however, the most important feature of duality is that, by applying the various duality maps, one can construct spectra of electric, magnetic and dyonic states, also represented by classical solutions of string theory [8]. Using the solutions/states correspondence, we compare the Bekenstein-Hawking entropy obtained from the area of the classical solution to the quantum-mechanical microcanonical counting of ensembles of states.

It turns out that solutions in string theory possess a very nice feature that greatly facilitates this comparison, namely, that of *compositeness*, whereby arbitrary solutions arise as bound states of single-charged fundamental constituents. For example, the Reissner-Nordström solution of Einstein-Maxwell theory arises in string theory as the composite of two pairs of electric and mag-

netic charges of various fields of ten-dimensional string theory compactified to four dimensions. To a distant observer, however, the composite appears as a single black hole.

Before proceeding with the entropy comparison for this black hole, let us return to the elementary particle/black hole correspondence to make sure we are on the right track. We first consider solutions which correspond to pure states, or which have zero entropy. Such solutions necessarily have zero area. It turns out that this is the case for those solutions which possess no more than two constituents. For the two cases, the quantum numbers (mass and charge) of the solutions match those of particular supersymmetric quantum string states. In addition, the dynamics of the black holes agree with the four-point amplitudes of the corresponding string states in the low-velocity limit [9]. For the single-charge black holes and their corresponding states, this scattering is trivial, while for the two-charge black holes and their corresponding states, we obtain Rutherford scattering. Of course this quantum number matching and dynamical agreement does not mean we can go ahead and identify the black hole solutions with elementary string states, but at least the correspondence makes sense.

Let us now return to our black hole with $S \neq 0$, or $A \neq 0$. This solution should correspond to an *ensemble* of string states. Now the laws of black hole thermodynamics follow from classical general relativity. However, the laws of thermodynamics in general follow from a microcanonical counting of quantum states, *i.e.*, from statistical mechanics. An important test of a theory of quantum gravity is then the following: can one obtain the black hole laws of thermodynamics from a counting of microscopic quantum states, *i.e.*, is there a quantum/statistical mechanical basis for these classical laws? We are interested in performing this test for string theory, where we have established a correspondence between classical solutions and elementary and solitonic states. On the one side, we can construct a black hole solution, compute its area and deduce the entropy from $S = A/4G$. On the other side, we can set up the ensemble of states corresponding to this solution, compute its degeneracy and take the logarithm to obtain the entropy.

As we have already mentioned, the Reissner-Nordström black hole is the bound state of four constituent single-charge black holes. Let g_{MN} and B_{MN} be the spacetime metric and antisymmetric tensor. The four charges corresponding to the four constituent black holes are given by (normalized to represent quantum-mechanical number operators): Q_e, electric with respect to $B_{\mu\nu}$, Q_m, magnetic with respect to $B_{\mu\nu}$, N_e, electric with respect to $g_{5\mu}$ and N_m, magnetic with respect to $g_{4\mu}$, where x_4 and x_5 are two compactified directions. Let us also simplify the picture slightly by setting $N_m = 1$.

The classical solution has a nonzero area given by $A = 8\pi G\sqrt{Q_e Q_m N_e}$, hence a Bekenstein-Hawking entropy $S_{BH} = 2\pi\sqrt{Q_e Q_m N_e}$. The setup of the corresponding string states is the following. We are interested in the case of large charges, corresponding to black hole solutions. For large N_e, the number operator N_e represents the momentum of massless open strings going between Q_e electric charges and Q_m magnetic charges. The total number of bosonic modes is then given by $4Q_e Q_m$, since there is a degeneracy associated with the extra (6789) part of the compactified space. By supersymmetry, the number of fermionic modes is also $4Q_e Q_m$. This system is then like a $1+1$-dimensional gas of massless left moving particles with $4Q_e Q_m$ bosonic and fermionic species of particles carrying total energy N_e/R, where R is the radius of the circle. The number of such modes is given by $d(N) \sim \exp 2\pi\sqrt{Q_e Q_m N_e}$, so that $S = \ln d(N) = 2\pi\sqrt{Q_e Q_m N_e} = S_{BH}$, in agreement with the area law. This is very exciting, as it is the first time we can derive the area law from a quantum-mechanical theory (string theory). This result was first found for five-dimensional extremal black holes [10] and subsequently for four-dimensional extremal black holes [11]. Analogous results for near-extremal black holes were also obtained [12], which seems to indicate that this sort of factorization is not a property of supersymmetry alone, although it is only for supersymmetric solutions that one can invoke non-renormalization theorems to protect the counting of states in going from the perturbative state-counting picture to the nonperturbative black hole picture.

The correspondence between black holes and string states can be understood as follows [13]. String states at level N have mass $M_s \sim N/l_s^2$ and entropy $S_s \sim \sqrt{N}$, where l_s is the string scale. This picture is valid provided the string coupling $g << 1$, where g is related to Newton's constant G in four dimensions via $G \sim g^2 l_s^2$. Now for the black hole solution of the low-energy field theory limit of string theory, the mass and entropy are given by $M_{BH} \sim R_S/G$ and $S_{BH} \sim R_S^2/G$, where R_S is the Schwarzschild radius. The mass and entropy of the string states become of the same order precisely when $R_S \sim l_s$, *i.e.* when the string scale becomes of the order of the Schwarzschild radius. This happens when $g \sim N^{-1/4}$. For N very large, g is still very small at this point. It then makes sense to refer to $\tilde{g} = gN^{1/4}$ as our effective coupling. For $\tilde{g} << 1$, we have perturbative string states. At $\tilde{g} \sim 1$, the black hole forms, the low-energy solution still being valid since g is still small. As we continue to turn up the coupling, the black hole picture continues to hold until we reach the region $g >> 1$, in which case a strong/weak coupling duality map presumably takes the black hole back into a perturbative state in the weak coupling limit of another string theory.

Of course we still do not understand the precise mechanism by which an

ensemble of states turns into a black hole. This and other questions remain to be answered, as well as the formulation of a duality-manifest string theory. In this regard, connections with the better-understood Yang-Mills [14] duality seem most promising.

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